Research Article

Google and the Mind

Predicting Fluency With PageRank

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ABSTRACT—***Human memory and Internet search engines face a shared computational problem, needing to retrieve stored pieces of information in response to a query. We explored whether they employ similar solutions, testing whether we could predict human performance on a fluency task using PageRank, a component of the Google search engine. In this task, people were shown a letter of the alphabet and asked to name the first word beginning with that letter that came to mind. We show that PageRank, computed on a semantic network constructed from word-association data, outperformed word frequency and the number of words for which a word is named as an associate as a predictor of the words that people produced in this task. We identify two simple process models that could support this apparent correspondence between human memory and Internet search, and relate our results to previous rational models of memory.***

Rational models of cognition explain human behavior as approximating optimal solutions to the computational problems posed by the environment (Anderson, 1990; Chater & Oaksford, 1999; Marr, 1982; Oaksford & Chater, 1998). Rational models have been developed for several aspects of cognition, including memory (Anderson, 1990; Griffiths, Steyvers, & Tenenbaum, 2007; Shiffrin & Steyvers, 1997), reasoning (Oaksford & Chater, 1994), generalization (Shepard, 1987; Tenenbaum & Griffiths, 2001), categorization (Anderson, 1990; Ashby & Alfonso-Reese, 1995), and causal induction (Anderson, 1990; Griffiths & Tenenbaum, 2005). By emphasizing the computational problems underlying cognition, rational models sometimes reveal connections between human behavior and that of other systems that solve similar problems. For example, Anderson's (1990; Anderson & Milson, 1989) rational analysis of memory identified parallels between the problem solved by human memory and that addressed by automated information-retrieval systems, arguing for similar solutions to the two problems. Since Anderson's analysis, information-retrieval systems have evolved to produce what might be an even more compelling metaphor for human memory—the Internet search engine—and computer scientists have developed new algorithms for solving the problem of pulling relevant facts from large databases. In this article, we explore the correspondence between these new algorithms and the structure of human memory. Specifically, we show that PageRank (Page, Brin, Motwani, & Winograd, 1998), one of the key components of the Google search engine, predicts human responses in a fluency task.

Viewed abstractly, the World Wide Web forms a directed graph, in which the nodes are Web pages and the links between those nodes are hyperlinks, as shown in Figure 1a. The goal of an Internet search engine is to retrieve an ordered list of pages that are relevant to a particular query. Typically, this is done by identifying all pages that contain the words that appear in the query, then ordering those pages using a measure of their importance based on their link structure. Many psychological theories view human memory as solving a similar problem: retrieving the items in a stored set that are likely to be relevant to a query. The targets of retrieval are facts, concepts, or words, rather than Web pages, but these pieces of information are often assumed to be connected to one another in a way similar to the way in which Web pages are connected. In an associative semantic network, such as that shown in Figure 1b, a set of words or concepts is represented using nodes connected by links that indicate pair-wise associations (e.g., Collins & Loftus, 1975). Analyses of semantic networks estimated from human behavior reveal that these networks have properties similar to those of the World Wide Web, such as a "scale-free" distribution for the number of nodes to which a node is connected (Steyvers & Tenenbaum, 2005). If one takes such a network to be the representation of the knowledge on which retrieval processes operate, human memory and Internet search engines address the same computational problem: identifying those items that are

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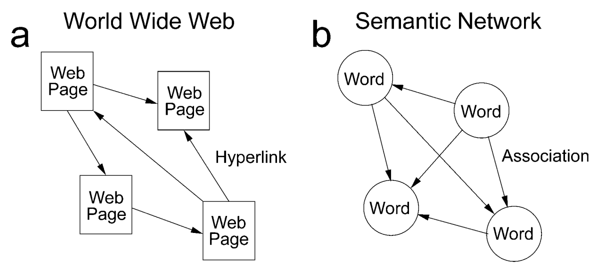

Fig. 1. **Parallels between the problems faced by search engines and human memory. Internet search and retrieval from memory both involve finding the items relevant to a query from within a large network of interconnected pieces of information. In the case of Internet search (a), the items to be retrieved are Web pages connected by hyperlinks. When items are retrieved from a semantic network (b), the items are words or concepts connected by associative links.**

relevant to a query from a large network of interconnected pieces of information. Consequently, it seems possible that they solve this problem similarly.

Although the details of the algorithms used by commercial search engines are proprietary, the basic principles behind the PageRank algorithm, part of the Google search engine, are public knowledge (Page et al., 1998). The algorithm makes use of two key ideas: first, that links between Web pages provide information about their importance, and second, that the relationship between importance and linking is recursive. Given an ordered set of n pages, we can summarize the links between them with an $n \times n$ matrix $\mathbf{L}$, where $\mathbf{L}_{ij}$ is 1 if there is a link from Web page j to Web page i and is 0 otherwise. If we assume that links are chosen in such a way that more important pages receive more links, then the number of links that a Web page receives (in graph-theoretic terms, its *in-degree*) could be used as a simple index of its importance. Using the n-dimensional vector $\mathbf{p}$ to summarize the importance of our n Web pages, this is the assumption that $\mathbf{p} = \mathbf{L1}$, where $\mathbf{1}$ is a column vector with n elements each equal to 1.

PageRank goes beyond this simple measure of the importance of a Web page by observing that a link from an important Web page is a better indicator of importance than a link from an unimportant Web page. Under such a view, an important Web page is one that receives many links from other important Web pages.[1] We might thus imagine importance as flowing along the links of the graph shown in Figure 1a. If each Web page distributes its importance uniformly over its outgoing links, then we can express the proportion of the importance of each Web page traveling along each link in a matrix $\mathbf{M}$, where $\mathbf{M}_{ij} = \mathbf{L}_{ij} / \sum_{k=1}^{n} \mathbf{L}_{kj}$. The idea that highly important Web pages receive links from highly important Web pages implies a recursive definition of importance, resulting in the equation

$$\mathbf{p} = \mathbf{Mp}, \tag{1}$$

which identifies $\mathbf{p}$ as the eigenvector of the matrix $\mathbf{M}$ with the greatest eigenvalue.[2] The PageRank algorithm computes the importance of a Web page by finding a vector $\mathbf{p}$ that satisfies this equation.

The empirical success of Google suggests that PageRank constitutes an effective solution to the problem of Internet search. This raises the possibility that computing a similar quantity for a semantic network might predict which items are particularly prominent in human memory. If one constructs a semantic network from word-association norms, placing links from words used as cues in an association task to the words that are named as their associates, the in-degree of a node indicates the number of words for which the corresponding word is produced as an associate. This kind of "associate frequency" is a natural predictor of the prominence of words in memory, and has been used as such in a number of studies (McEvoy, Nelson, & Komatsu, 1999; Nelson, Dyrdal, & Goodmon, 2005). However, this simple measure assumes that all cues should be given equal weight, whereas computing PageRank for a semantic network takes into account the fact that the cues themselves differ in their prominence in memory.

To explore the correspondence between PageRank and human memory, we used a task that closely parallels the formal structure of Internet search. In this task, we showed people a letter of the alphabet (the query) and asked them to say the first word beginning with that letter that came to mind. By using a query that was either true or false of each item in memory, we aimed to mimic the problem solved by Internet search engines, which retrieve all pages containing the set of search terms, and thus to obtain a direct estimate of the prominence of different words in human memory. In memory research, such a task is used to measure *fluency*—the ease with which people retrieve different facts. This particular task is used to measure letter fluency or verbal fluency in neuropsychology, and has been applied in the diagnosis of a variety of neurological and neuropsychiatric disorders (e.g., Lezak, 1995). However, in the standard use of this task, the interest is in the number of words that can be produced in a given time period, whereas we intended to discover which words were more likely to be produced than others.

Our goal was to determine whether people's responses in this fluency task were better predicted by PageRank or by more conventional predictors—word frequency and associate fre-

[1]A similar insight in the context of bibliometrics motivated the development of essentially the same method for measuring the importance of publications linked by citations (Geller, 1978; Pinski & Narin, 1976).

[2]In general, an eigenvector $\mathbf{x}$ of a matrix $\mathbf{M}$ satisfies the equation $\lambda\mathbf{x} = \mathbf{Mx}$, where λ is the eigenvalue associated with $\mathbf{x}$ (Strang, 1988). Equation 1 identifies $\mathbf{p}$ as an eigenvector of $\mathbf{M}$ with eigenvalue 1. This is guaranteed to be the eigenvector with greatest eigenvalue because $\mathbf{M}$ is a stochastic matrix, with $\sum_{i=1}^{n} \mathbf{M}_{ij} = 1$, and thus has no eigenvalues greater than 1. For simplicity, all of the mathematical results reported in this article assume that $\mathbf{M}$ has only one eigenvalue equal to 1. The PageRank algorithm can be modified when this assumption is violated (Brin & Page, 1998), and a similar correction can extend the results we state here to the general case.

quency. Word frequency is viewed as a cause of fluency (e.g., Balota & Spieler, 1999; Plaut, McClelland, Seidenberg, & Patterson, 1996; Seidenberg & McClelland, 1989; see also Adelman, Brown, & Quesada, 2006) and is used to set the prior probability of items in rational models (Anderson, 1990; D. Norris, 2006). Associate frequency was computed from the same data as PageRank, differing only in the assumption that all cues should be given equal weight. These two measures thus constitute strong alternatives to compare with PageRank.

METHOD

Fifty members of the Brown University community (30 female, 20 male) participated in the experiment. Their ages ranged from 18 to 75 years, with a mean of 24.6 years and a standard deviation of 13.2 years.

Twenty-one letters of the alphabet (the low-frequency letters *K*, *Q*, *X*, *Y*, and *Z* being excluded) were printed individually on 3 × 5 cards in 56-point Times New Roman font. The cards were shuffled, face down, and each subject was told that he or she would be shown letters of the alphabet one after the other and should produce the first word beginning with each letter that came to mind. The experimenter then turned the cards up one by one until the subject had responded to the entire set. The experimenter wrote down the words produced by the subject. This procedure was performed twice with each subject.

RESULTS

PageRank and associate frequency were calculated using a semantic network constructed from the word-association norms of Nelson, McEvoy, and Schreiber (1998). The norms list all words named at least twice as an associate of each of 5,018 words. From these norms, we constructed a directed graph in which each word was a node, with links to its associates. We then applied the PageRank algorithm to this graph and also calculated the associate frequency for each word. Finally, we recorded from Kucera and Francis (1967) the word frequency for each of the words appearing in the norms.

PageRank, associate frequency, and word frequency all define a ranking of the words that people could produce in our task. We used the responses of our subjects to evaluate these predictors. Responses that did not begin with the appropriate letter were omitted, as were instances of repetition of a word by a single subject, as these words could have been produced as a result of memory for the previous trial. The number of times each word was produced was then summed over all subjects. Table 1 shows the most popular responses for seven of the letters. For evaluating the predictors, we removed words that were produced only once. This is a standard procedure used to control outliers in tasks generating spontaneous productions, such as word-association tasks (including that of Nelson et al., 1998). Finally, we omitted all responses that did not appear in the word-association norms of Nelson et al., as PageRank and associate frequency were restricted to these words. The result was a set of 1,017 responses.

Our analysis focused on the ranks that the predictors assigned to the human responses. We identified all words in the norms that began with each letter and then ordered those words by each predictor, assigning a rank of 1 to the highest-scoring word and lower rank (i.e., a higher number) as the score decreased. Table 1 shows some of these ranks. Because the total number of words in the norms varied across letters (from 648 for *S* to 50 for *J*), we reduced these ranks to percentages of the set of possible responses for each letter before aggregating across letters. The distribution of ranks was heavily skewed, so we compared the predictors using medians and nonparametric tests. The median percentile ranks for the different predictors are shown in Table 2. Figure 2 presents the proportion of human responses produced as a function of percentile rank. PageRank outperformed both associate frequency and word frequency as a predictor of fluency, assigning lower ranks to 59.42% and 81.97%, respectively, of the human responses given different ranks by the different predictors (both ps < .0001 by binomial test).

We performed several additional analyses in an attempt to gain insight into the relatively poor performance of word frequency as a predictor. First, we used word frequencies from a larger corpus—the Touchstone Applied Science Associates (TASA) corpus used by Landauer and Dumais (1997)—which improved predictions, although not enough for word frequency to compete with PageRank or associate frequency. Second, we looked at performance within restricted sets of words. Although the words used in our analyses were all produced as associates in the word-association task of Nelson et al. (1998), they varied in part of speech and concreteness. From Table 1, it is apparent that people's responses in the fluency task were biased toward concrete nouns, whereas word frequency does not take part of speech or concreteness into account. We repeated our analysis using two subsets of the words from the norms. The first subset consisted of all words identified as nouns and possessing concreteness ratings in the MRC Psycholinguistic Database (Wilson, 1988). This reduced the total number of words to 2,128, and the total number of responses matching these words to 753. This restriction controlled for part of speech, but still left differences in concreteness among the words favored by the predictors; the mean concreteness ratings averaged over the distributions over words implied by PageRank, associate frequency, and word frequency from the Kucera and Francis (1967) and TASA corpora were 496.93, 490.32, 421.24, and 433.65, respectively (on a scale from 100 to 700). To address this issue, we also analyzed a more reduced subset of words: nouns with concreteness ratings greater than or equal to the median. This second subset included 1,068 words matching 526 human responses and had mean concreteness ratings of 584.70, 583.93,

TABLE 1

Human Subjects' Responses in the Fluency Task and Rankings Given by the Predictors

Beginning letter						
A	B	C	D	P	S	T
			Human responses			
Apple (25)	Boy (11)	Cat (26)	Dog (19)	People (5)	Snake (11)	Tea (5)
Alphabet (7)	Bat (6)	Car (8)	Dad (16)	Penguin (3)	Stop (4)	Television (5)
Ant (6)	Banana (5)	Cool (3)	Door (5)	Pizza (3)	Saw (2)	Time (4)
Aardvark (3)	Balloon (4)	Card (2)	Down (4)	Play (3)	Sea (2)	Tree (4)
Ace (2)	Book (4)	Class (2)	Dark (3)	Pop (3)	Sex (2)	Table (3)
Ambulance (2)	Baby (3)	Coke (2)	Dumb (3)	Puppy (3)	Silly (2)	Tall (3)
Animal (2)	Ball (2)	Cookie (2)	Day (2)	Piano (2)	Sister (2)	Tank (3)
Absence (1)	Barn (2)	Crack (2)	Devil (2)	Pie (2)	Sit (2)	Telephone (3)
Acrobat (1)	Bear (2)	Cross (2)	Dinosaur (2)	Pig (2)	Slither (2)	Town (3)
Act (1)	Beef (2)	Cut (2)	Do (2)	Power (2)	South (2)	Train (3)
			PageRank			
Animal (2)	Big (0)	Cold (0)	Dog (19)	Pretty (0)	Small (1)	Time (4)
Away (0)	Bad (1)	Car (8)	Dark (3)	People (5)	Sad (1)	Tall (3)
Air (0)	Boy (11)	Cat (26)	Drink (1)	Paper (0)	School (0)	Talk (1)
Alone (0)	Black (0)	Color (0)	Down (4)	Pain (0)	Sun (2)	Tree (4)
Apple (25)	Beautiful (0)	Clothes (0)	Death (1)	Puppy (3)	Smile (0)	Tired (0)
Arm (0)	Blue (2)	Child (1)	Door (5)	Person (1)	Stop (4)	Tiny (0)
Ache (0)	Book (4)	Cute (0)	Day (2)	Play (3)	Soft (1)	Thin (0)
Answer (1)	Body (0)	Clean (0)	Dirty (0)	Place (1)	Sex (2)	Top (1)
Apartment (0)	Bright (0)	Close (0)	Dirt (0)	Party (0)	Sky (0)	Together (0)
Alcohol (0)	Baby (3)	Cry (0)	Dead (0)	Pen (0)	Sleep (0)	Train (3)
			Associate frequency			
Animal (2)	Bad (1)	Car (8)	Dog (19)	Paper (0)	School (0)	Time (4)
Air (0)	Book (4)	Clothes (0)	Death (1)	Pain (0)	Small (1)	Tree (4)
Army (0)	Black (0)	Cold (0)	Drink (1)	People (5)	Sex (2)	Talk (1)
Away (0)	Big (0)	Clean (0)	Dirty (0)	Person (1)	Sad (1)	Together (0)
Anger (0)	Baby (3)	Child (1)	Dark (3)	Play (3)	Soft (1)	Test (1)
Answer (1)	Ball (2)	Class (2)	Down (4)	Party (0)	Stop (4)	Television (5)
Art (0)	Body (0)	Church (0)	Dirt (0)	Pretty (0)	Smell (0)	Think (0)
Apple (25)	Bird (0)	Cut (2)	Dead (0)	Problem (0)	Strong (0)	Top (1)
Alcohol (0)	Break (0)	Color (0)	Dance (0)	Police (1)	Smart (0)	Teacher (0)
Arm (0)	Boring (0)	Cat (26)	Danger (1)	Place (1)	Sick (0)	Take (0)
			Word frequency			
A (0)	Be (1)	Can (0)	Do (2)	People (5)	She (0)	There (0)
All (0)	Before (0)	Come (0)	Down (4)	Place (1)	Some (0)	Than (0)
After (1)	Back (0)	Course (0)	Day (2)	Part (0)	State (1)	Time (4)
Another (0)	Because (0)	City (0)	Development (0)	Public (1)	Still (0)	Two (1)
Against (0)	Between (0)	Case (0)	Done (1)	Put (2)	See (0)	Through (0)
Again (0)	Being (0)	Children (0)	Different (0)	Point (0)	Same (0)	Take (0)
American (0)	Better (0)	Church (0)	Door (5)	Program (0)	Since (0)	Three (0)
Around (0)	Business (0)	Country (0)	Death (1)	President (0)	Small (1)	Thought (0)
Always (0)	Become (0)	Certain (0)	Department (0)	Present (0)	Say (1)	Think (0)
Away (0)	Big (0)	Company (0)	Dark (3)	Possible (0)	School (0)	Thing (0)

Note. This table provides a selective list, showing only 10 items for each letter. In the sections of the table corresponding to the three predictors, the order of the words in each column reflects the rankings given by the predictor indicated. Numbers in parentheses are frequencies in the human responses. Only responses that were produced at least twice were used in the comparison of models, as a means of controlling for outliers.

574.73, and 577.31, respectively. PageRank still consistently outperformed the other predictors for these two restricted subsets of words, as shown in Table 2 and Figure 2.

Finally, although PageRank is typically computed purely from link structure, the word-association norms also provided us with information about the probability with which associates were

TABLE 2
Median Percentile Ranks Assigned to the Human Responses by Different Predictors

Predictor	All words	Nouns only	Concrete nouns only
PageRank	8.33[a]	8.16[b]	13.33[c]
Associate frequency	10.00	14.77	17.54
Word frequency: KF	29.09	36.54	33.33
Word frequency: TASA	18.99	22.51	21.64
Weighted PageRank	7.14	8.57[b]	13.33[c]
Weighted associate frequency	8.24[a]	12.93	16.67

Note. All pair-wise differences within each column are statistically significant at $p < .01$ (two-sided paired Wilcoxon signed rank tests), except as indicated by superscripts: [a]$p = .051$, [b]$p = .023$, and [c]$p = .852$. KF = word frequencies from Kucera and Francis (1967); TASA = word frequencies from Landauer and Dumais (1997).

produced. We defined a matrix **M** that used these probabilities, rather than assuming that outgoing links were selected uniformly at random. The PageRank algorithm could still be applied to this matrix, although the result no longer had a simple interpretation as the first eigenvector of the matrix, as people commonly produced associates outside the set covered by the norms (an equivalent issue arises with Web pages that create "dangling links" to other Web pages that do not link back into the target set; Page et al., 1998). We also computed a "weighted" measure of associate frequency, adding the probabilities with which people produced each word as an associate (i.e., taking $\mathbf{p} = \mathbf{M1}$). The weighted measures produced an improvement for both PageRank and associate frequency when results for all words were compared, as shown in Table 2 and Figure 2, and reduced the difference between these two predictors slightly, with PageRank assigning lower ranks to 54.82% of human responses ($p < .005$ by binomial test).

DISCUSSION

The results of our experiment indicate that PageRank, computed from a semantic network, is a good predictor of human responses in a fluency task. Specifically, PageRank outperformed two measures of the prominence of words in memory: word frequency and associate frequency. These results suggest that the PageRank of a word could be used in place of these measures when designing or modeling memory experiments, providing a new way to predict the prominence of items in memory from word-association data. If one assumes that the semantic network used to generate these predictions accurately captures the underlying representation, these results also support our hypothesis that human memory and Internet search engines might solve their shared problem in similar ways. In the remainder of this article, we identify some connections to existing cognitive models, describe some simple mechanisms that could produce a correspondence between PageRank and human memory, and clarify the relationship between our analysis and the approaches taken in rational models of memory.

Connections to Other Cognitive Models

The ideas behind PageRank are simple and appealing, so it is perhaps not surprising that there are at least two instances of similar cognitive models. First, Sloman, Love, and Ahn (1998) independently proposed using a method equivalent to PageRank to measure the centrality of features to concepts.[3] For Sloman et al., each entry in the vector **p** indicates the centrality of a particular feature, and each entry in the matrix **M** encodes the extent to which a given feature depends on another in a particular concept (e.g., the fact that robins have feathers depends on the fact that robins fly). The recursive model defined in Equation 1 provided good predictions of human judgments of feature centrality. Second, Steyvers, Shiffrin, and Nelson (2004) found that the distances between words in "word association spaces," constructed from word-association norms, predict human performance on a range of memory tasks. The dimensions of these word-association spaces correspond to the first few eigenvectors of the matrix giving the probability with which people named each word as an associate of another word—the weighted matrix **M** defined in our Results section. The first dimension of such a space thus corresponds closely to the weighted form of PageRank, with the only difference between these two measures resulting from the fact that PageRank also takes into account dangling links (Page et al., 1998).

Psychological Mechanisms That Might Produce the Correspondence With PageRank

Our observation of a correspondence between human memory and PageRank minimally provides an improved method for predicting the prominence of items in memory from word-association data. However, our results could potentially be explained as the result of some simple psychological mechanisms. Much research on semantic networks assumes that activation spreads from node to node along associative links (e.g., Anderson, 1983; Collins & Loftus, 1975). Let the vector $\mathbf{x}^{(t)}$ denote the activation of a set of nodes at time t. If we assume that each node spreads its activation equally over the nodes to which it has links (guaranteeing that the total amount of activation in the system is conserved), and that the activation of a node at time $t + 1$ is determined by a decayed version of its activation at time t and the sum of its inputs, we obtain

$$\mathbf{x}^{(t+1)} = \alpha\mathbf{x}^{(t)} + (1 - \alpha)\mathbf{M}\mathbf{x}^{(t)}, \qquad (2)$$

where α is a decay constant and **M** is the matrix defined in the introduction. The vector **p** defined by Equation 1 is the equi-

[3] We thank Josh Tenenbaum for pointing out this connection.

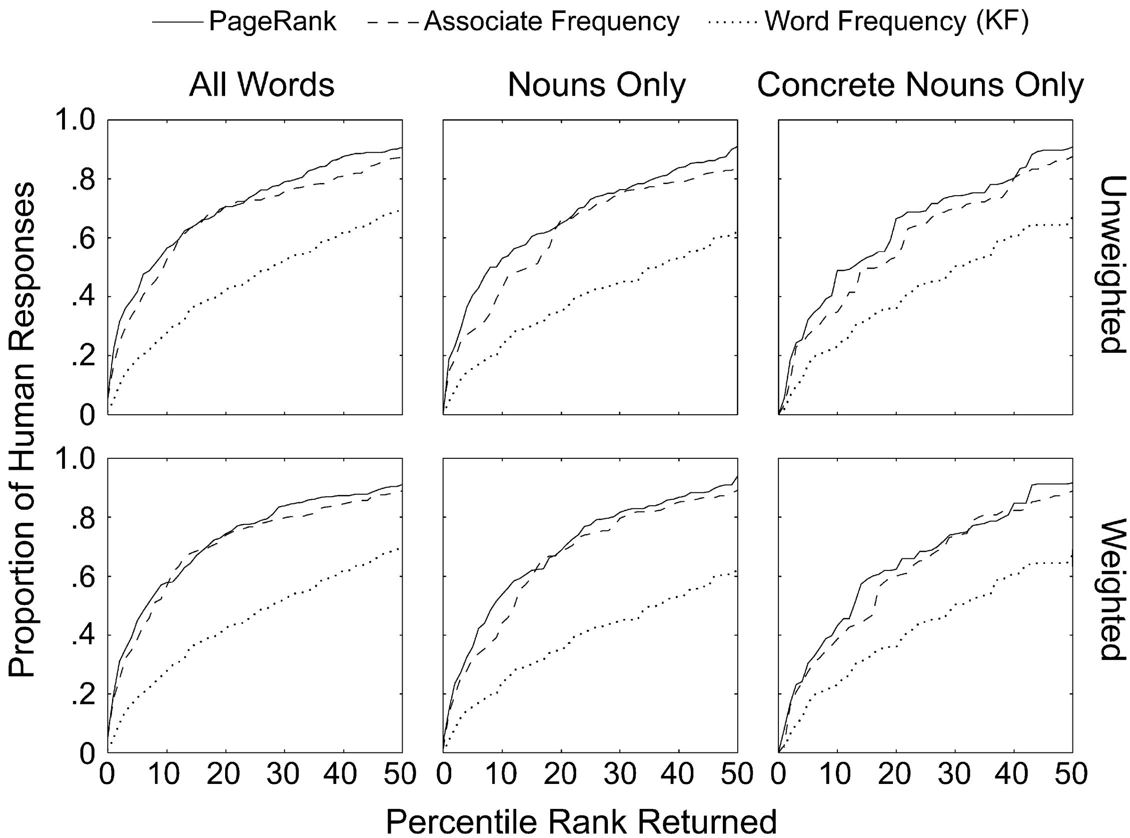

Fig. 2. Proportion of human responses correctly identified by each of three predictors, as a function of percentile rank. Curves closer to the top right-hand corner indicate better performance, and the percentile rank at which the proportion is .5 is the median reported in Table 2. The three predictors tested were PageRank, associate frequency, and word frequency from Kucera and Francis (1967; KF). PageRank and associate frequency were computed with two different methods: weighted (bottom row) and unweighted (top row). From left to right, results are shown for all words, nouns only, and nouns with concreteness scores greater than or equal to the median.

librium state of this system for any α less than 1, and $\mathbf{x}^{(t)}$ will converge to this state as t approaches infinity (Hirsch & Smale, 1974). Thus, the resting state of the semantic network would be one in which nodes receive activation in proportion to their PageRank. If higher activation results in a higher probability of retrieval, we should expect to see an effect of PageRank in human memory.

A similar argument applies to another simple model of our task. In attempting to account for the structure of people's "Trayne of Thoughts," Hobbes (1651/1998) suggested that one might move from notion to notion along paths of association. More formally, one might imagine that the particular words and concepts that are at the forefront of one's mind are produced by a random walk on a semantic network, with the transition from one node to another being made by choosing a link uniformly at random. This process defines a Markov chain, in which the state space is the nodes of the graph and the probability of moving between states is summarized in the matrix $\mathbf{M}$, which is known as the transition matrix. Regardless of a Markov chain's initial state, the probability that it is in a particular state converges to a fixed distribution (known as the *stationary distribution*) as the number of transitions increases (e.g., J.R. Norris, 1997). The stationary distribution of a Markov chain is a distribution that is invariant to multiplication by the transition matrix, meaning that taking a transition does not affect the probability of being in a particular state. This definition identifies the stationary distribution as the vector $\mathbf{p}$ that satisfies Equation 1, normalized so that $\sum_{i=1}^{n} \mathbf{p}_i = 1$. Consequently, the stationary distribution of a random walk on a semantic network will assign each node probability proportional to its PageRank. Equivalently, the PageRank of a Web page is the number of times it will be visited by a "random surfer" who clicks on links at random (Brin & Page, 1998; Page et al., 1998).

These properties of Markov chains have two implications for present purposes. First, the probability that a subject thinks of a particular word at a given moment will be proportional to the PageRank of that word, assuming that he or she has been thinking for long enough for the Markov chain to have converged to its stationary distribution. Second, if a subject then proceeds to search his or her memory by randomly following associative links until the subject finds a word that matches a query, the probability of selecting a particular word will be proportional to its PageRank, because the stationary distribution is invariant to further transitions. The optimal solution to the retrieval problem is to return the item with the highest PageRank, and a random walk on a semantic network approximates this solution by returning an item with probability proportional to its PageRank. This will be a good approximation when the distribution of PageRank is dominated by a few items with very high scores.

Relationship to Rational Models of Memory

Anderson's (1990; Anderson & Milson, 1989) rational model of memory formulates the problem of retrieval as one of statistical inference in which a set of hypotheses (which item in memory is needed) is evaluated in the light of data (the query). Such a problem can be solved by applying Bayes' rule. We can encode the probability with which a particular item h is likely to be needed in general with a prior probability distribution $P(h)$. If we use d to denote the data provided by a query, we want to find the posterior probability distribution $P(h|d)$. Bayes' rule indicates that

$$P(h \mid d) = \frac{P(d \mid h)P(h)}{\sum\limits_{h' \in H} P(d \mid h')P(h')}, \tag{3}$$

where the likelihood $P(d|h)$ indicates the probability that we would have observed d if h were the item needed, and H is the set of all hypotheses—in this case, one for each piece of information. The posterior distribution gives the probability that each item was the one sought in the query, and the optimal solution to the retrieval problem is to return pieces of information in decreasing order of their posterior probability.

Our approach to the problem of retrieval is entirely consistent with this Bayesian framework. In the case of Internet search, the items are Web pages, and the query is a string of words. Most search engines make the simplifying assumption that the likelihood $P(d|h)$ is constant for all Web pages that contain the words in the query and zero otherwise. Under this assumption, the optimal solution to the retrieval problem reduces to identifying all pages containing the query and ordering them by their prior probability. Thus, PageRank can be considered an estimate of the prior probability that a particular item is likely to be needed. The idea that this probability can be estimated from the links between items is complementary to the approach that has been taken in rational models of memory, which have emphasized the pattern of past usage as a source of these estimates (Anderson, 1990; Anderson & Milson, 1989; Anderson & Schooler, 1991; Schooler & Anderson, 1997). Priors based on link structures can be used to answer other kinds of queries—for example, a search may be for an associate of a word rather than a word beginning with a particular letter—by using a more sophisticated form for the likelihood $P(d|h)$. The models introduced by Anderson and his colleagues provide an account of how the likelihood should be computed for different kinds of items (see also Griffiths et al., 2007).

CONCLUSION

The relationship between PageRank and fluency reported in this article suggests that the analogy between computer-based solutions to information retrieval problems and human memory may be worth pursuing further. In particular, our approach indicates how one can obtain novel models of human memory by studying the properties of successful information-retrieval systems, such as Internet search engines. Establishing this correspondence is important not just for the hypotheses about human cognition that may result, but as a path toward developing better search engines. For example, our discussion of the relationship between rational models of memory and Internet search highlights two areas in which human memory research might extend the capacities of search engines: by providing an account of how to go beyond simple matching of the words contained in a query when defining the probability of that query given a Web page, and by indicating how information about past usage can be combined with link structure. These problems are actively being explored in computer science, but the parallels between Internet search and human memory suggest that one might be equally likely to find good solutions by studying the mind.

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