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A MODIFIED DUTCH BOOK ARGUMENT

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A unifying strand in the debate between objectivists and subjectivists is the thesis that a man's degrees of belief *ought* to obey the axioms of the probability calculus. This paper is concerned to reconstruct the argument most widely employed in support of this thesis – the Dutch Book Argument (DBA). We first note a critical shortcoming in the usual presentation of the DBA; we then introduce a general principle of rationality as the basis for a modification that overcomes this shortcoming; and, finally, we argue that the modified DBA is immune from certain further objections that might be or have been advanced to the DBA.

I. THE STANDARD DBA

As standardly presented, the DBA has three parts: a part relating degree of belief and betting behaviour, a part concerned with the rationality and coherence of sets of bets, and a formal part giving mathematical demonstrations that certain sets of bets have certain properties. The formal part is not in dispute, and will be presupposed in our discussion.

We take it that there is such a thing as partial belief, and that it is conceptually related to betting behaviour, and describe the latter more or less standardly thus: A bet with respect to p is specified by (i) a (positive) sum of money, S, the *stake*, (ii) a number between 0 and 1, Qp, the *betting quotient*, and (iii) whether the bet is *on* p or *against* p (the direction of the bet). A bet on p at quotient Qp involves an outlay (=possible loss) of $S \cdot Qp$ and a win, if p turns out true, of $S - S \cdot Qp$. A bet against p involves outlay of $S - S \cdot Qp$ and a win, if p turns out false, of $S \cdot Qp$. For concreteness, the bets discussed below are treated in the usual way as money bets. We assume that it is desirable to win money and undesirable to lose money. (The justification of the assumption is discussed later.)

Finally, we take it that the betting quotient to be identified with a person's degree of belief in p is his *forced* quotient (Forc Qp).[1] The person is compelled to nominate a quotient prior to being informed whether he will be betting on or against p at that rate.

Now for the critical shortcoming.[2] How might a dutch book style argument justify the forced betting quotient correlate of, say, the Negation Axiom, namely

$$\text{Forc}\,Qp + \text{Forc}\,Q{\sim}p = 1 \tag{1}$$

The essential insight about rationality that underlies the DBA is that a course of action that must lead to nett loss is irrational. But how does violating (1) lead to a certain loss? There are two cases: the sum is greater than one, the sum is less than one. Now if $\text{Forc}\,Qp + \text{Forc}\,Q{\sim}p > 1$, bets *on* p and ${\sim}p$ with equal stake lead to a certain loss; but bets *against* p and ${\sim}p$ with equal stake lead to a certain *gain*. If $\text{Forc}\,Qp + \text{Forc}\,Qp < 1$, bets *against* p and ${\sim}p$ lead to a certain loss; but bets *on* p and ${\sim}p$ lead to a certain gain.[3]

It is hard to connect this with the essential insight about rationality. True, violating (1) *and* betting in a certain way leads to a loss come what may; but this, following the insight, only shows the conjunction irrational. And this surely is insufficient *in itself* to show that it is irrational to violate (1). After all, violating (1) and betting in a different way leads to a certain gain.

The fact is that the connection between irrationality and betting quotients which do not obey the probability axioms, has not been made sufficiently clear to sustain the DBA. As a first step towards a repair we state a general principle governing rationality.

II. RATIONALITY AND UNIVERSALIZABILITY

If act A is (morally) right and B is like A in all relevant respects, then B is right. This principle, where 'right' may be replaced by 'good', 'bad', 'ought to be done', and so on, is the so-called universalizability principle in Ethics. A similar principle applies to rational belief. If it is rational to believe proposition p in situation S_1, and situation S_2 is, in all respects relevant to the *truth* of p, like S_1, then it is rational to believe p in S_2. We take this principle to be an evident truth – one such that the best argument for it is to state it.

The corresponding principle for degrees of rational belief is: If it is rational to believe p in S_1 to degree R and S_2 is identical to S_1 in all respects relevant to the truth of p, then it is rational to believe p in S_2 to degree R.

III. A RECONSTRUCTED DBA

Consider a person, X, who is forced to bet concerning propositions, $p_1, \ldots, p_n$, under the following conditions: (i) the direction of his bet regarding each p_i is determined by a person, Y, who knows his betting quotient, Q_i, for each p_i, (ii) X specifies these betting quotients in ignorance of the direction of his bets, (iii) Y wants X to lose, and (iv) Y knows the formal part of the DBA. The quotients here will be forced quotients, and so Q_i may be identified with X's degree of belief in p_i for each i. We will call a situation satisfying (i)–(iv), a competitive betting situation; and quotients in such situations, competitive quotients.

Now it *will* be paradigmatically irrational for X to have a set of competitive betting quotients which do not obey the probability calculus. Because Y wants X to lose and, as he knows the formal part of the DBA, Y can ensure that X does lose.

Therefore, a set of competitive betting quotients which violate the probability axioms are irrational. But these betting quotients *are* degrees of belief: they are determined by degrees of belief,[4] so if the former are irrational, so are the latter. Hence, it is irrational to have a set of degrees of belief which violate the probability axioms *in a competitive situation* (as defined in (i)–(iv), above).

We now eliminate this italicized proviso by reference to the universalizability principle for rational belief. Consider the normal situation where someone believes $p_1, \ldots, p_n$ to degrees $r_1, \ldots, r_n$, respectively, but is not going to be betting about them. Call this situation S_n. Now consider S_c, a situation exactly like S_n in every respect relevant to the *truth* of $p_1, \ldots, p_n$, and differing just in that clauses (i)–(iv) of the competitive betting situation apply. Therefore, $r_1, \ldots, r_n$ in S_c should be identical to $r_1, \ldots, r_n$ inS_n, by universalizability. But, as shown, the former ought to obey the probability axioms; so if the latter should be identical with the former, then so ought the latter.

Clearly, this kind of universalizability argument is always possible. For *any* situation in which one believes a set of propositions to varying degrees, there is a situation identical in all truth-relevant respects, which fulfils the conditions defining a competitive betting situation. In the latter, the DBA shows that a necessary condition of one's degrees of belief being rational is concordance with the probability axioms. And, by universalizability, one's degrees of belief ought to be the same in both situations. Hence, our modified DBA enables us to extend this necessary condition to any belief-situation whatever.

IV. ON TWO OBJECTIONS

We now show how two serious objections, further to the 'critical shortcoming' of § 1, to the standard DBA can be handled within the framework of our reconstruction.

(i) *The Person Without Desire*

The equation of money with the desirable in the DBA is standardly alleged to be a harmless assumption of convenience. But it conceals a substantive assumption – that the person in question has desires. And surely a person *might* have beliefs without desires. For such a person, the standard DBA cannot even get started.

In our approach, we imagine such a person endowed (perhaps miraculously) with desires in a manner irrelevant to the truth of the propositions in question; show that such an endowed person's degrees of belief ought to obey the axioms; and then extend the result to the original person by reference to universalizability.

(ii) *Unsettled Bets*

In real life some bets can only be settled one way – for instance, a bet on there being three successive sevens in the decimal expansion of π can only be settled affirmatively; and other bets cannot be settled either way – for instance, one on Julius Caesar having winked ten times crossing the Rubicon. This gives the standard DBA difficulty, for in such cases rational betting quotients need not obey the axioms.[5]

In our approach, we imagine that God settles all such bets, and this is known to both parties. And then argue in the manner of two paragraphs back using universalizability.

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NOTES

[1] This assumption is not crucial in what follows, and readers who favour some other quotient, e.g., the maximum quotient on p, may insert their favourite throughout what follows.

[2] We put it briefly as it is well put in Patricia Baillie, 'Confirmation and The Dutch Book Argument', *British Journal for The Philosophy of Science* **24** (1973), 393–397. It was, indeed, her paper that convinced us of the need to modify the DBA.

[3] In accord with the policy of omitting the formal part of the DBA, we leave out the arithmetic here.

[4] This is not, of course, true in non-competitive situations where Y wants X to *win*.

[5] For details see Ian Hacking, 'On Falling Short of Strict Coherence', *Philosophy of Science* **35** (1968), 284–286; and B. D. Ellis, 'The Logic of Subjective Probability', *British Journal for the Philosophy of Science* **24** (1973), 125–152.